Improved Semi parametric Regression Model
based on Sparse Representation for LEO Orbit Determination

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1. Abstract

In satellite orbit determination model, several kinds of nonlinear components are superposed on the principal linear part, including:

- the color-noise of observational data
- the dynamic modeling error
- the truncation error
Traditional method

- Generally, the principal linear signal is estimated by Least Square or Kalman filtering, while the nonlinear part is estimated by expanding them on some base. But the estimate precision depends on whether the selected base matches the characteristic of the nonlinear error very well.
Our novel method

- to estimate the nonlinear components based on the technique of sparse signal representation in an over-complete dictionary, it can provide a reasonable sparse representation of the error with high precision.

- an orbit determination model based on improved semi-parametric regression is constructed, in which the dynamic model error, orbit perturbation and the observation residual are represented on a well-designed dictionary.
2. Introduction

Precise orbit determination means that an “integrated” model is introduced to improve the orbit estimation precision by combining:

- the observation data
- dynamical model
- appropriate nonlinear parametric algorithm

Therefore, the problem of orbit determination can be regarded as a nonlinear multi-model estimation problem.
Nonlinear factors

- The undiscovered factors or error in dynamical model.
- The truncation error of linearization of observation equation and state equation.
- The systematic error and random error in observation data.
General Solution method

- the outliers dealing method
- Gauss-Markov process noise model application
- some basis function approximation
- the semiparametric model
Our semiparametric method

- the dynamic model error is estimated by iterative frequency character analysis and trend fitting
- the observation residual is represented on a well-designed dictionary
- the semi-parametric iterative algorithm
3. Orbit Determination model based on semiparametric regression for BPS

3.1 dynamical model

- According to the Newton’s Second Law, satellites’ dynamics model can be described as follows:

\[
\ddot{\mathbf{r}} = f_{TB} + f_{NB} + f_{NS} + f_{TD} + f_{RL} + f_{SR} + f_{AL} + f_{DG} + f_{TH}
\]
Complex Perturbation

With the improving requirement of orbit determination precision, the research of the perturbation forces also becomes finer. On the one hand, finer perturbation forces can enhance the orbit determination precision, while on the other hand, the complexity of orbit determination calculation will also be greatly increased.
The concept of perturbation based on SP

We seek for the optimal basis function and then erects a sparse parameters representation models of the orbit perturbation, which can be used to take the place of the complicated perturbation forced equations and consequently reduce the computational complexity.
The method of perturbation based on SP

According to our analysis for the orbit character and perturbation error, we give the representation for the difference between the all forces and main forces as follows:

\[
\Delta R = \sum_{m=0}^{M-1} \left( \sum_{k=0}^{K-1} a_k^{(m)} t^k \right) \cdot \exp\{j2\pi f_m t\}
\]

Where \( f_m (m = 0, \cdots, M - 1) \) are the frequencies of perturbation error, and \( a_k^{(m)} \) is the corresponding polynomial coefficients, and they are all parameters to be estimated.
The method of perturbation based on SP

the ESPRIT (Estimation of signal parameters via rotational invariance technique) method to estimate the frequencies, the model is:

\[ y(n) = \Delta R(n) + v(n) = \sum_{i=1}^{p} a_i(n)s_n(\omega_i) + v(n) \]

By solving the generalized eigenvalues we can determine the main frequencies with corresponding trend.
algorithm description

For the difference \( \Delta R \), estimate one of the main frequency \( f_0 \);

Shift the main frequency to zero point to obtain its trend, then estimate its trend parameter \( a_k^{(0)} \), \( k = 0, \cdots, K - 1 \) by select appropriate basis such as polynomial basis;

Remove the signal of main frequency \( f_0 \) from the deviation \( \Delta R \), denote it by \( \Delta R^{(1)} \). iterative computation, till the residual has no obvious low frequency of signal.
result

Fig. 1 Orbit position bias in x orientation formed by dynamical model error

Fig. 2 Orbit position residual in x orientation after ESPRIT iterative modeling
3.2 state equation

- According to the 2.1 section, the dynamical model of satellite should be written as

$$\begin{cases}
\dot{\bar{X}} = F(\bar{X}, t) + \Delta \ddot{R}(t) + \bar{w}(t) \\
X(t_0) = X_0
\end{cases}$$

- the state transition matrix solution:

$$\begin{cases}
d\Phi(t, t_0)/dt = A(t)\Phi(t, t_0) \\
\Phi(t_0, t_0) = I
\end{cases}$$
3.3 Observation equation

Measurement model based on BPS

\[
\begin{align*}
Y_1 &= |Sat_1 - C| := G_1(X(t), t) = \tilde{G}_1(X_0, t), \\
Y_2 &= |Sat_2 - C| := G_2(X(t), t) = \tilde{G}_2(X_0, t), \\
Y_3 &= 2(|Sat_1 - O| + |Sat_1 - U|) := G_3(X(t), t) = \tilde{G}_3(X_0, t), \\
Y_4 &= |Sat_1 - O| + |Sat_1 - U| + |Sat_2 - O| + |Sat_2 - U| \\
& := G_4(X(t), t) = \tilde{G}_4(X_0, t).
\end{align*}
\]

Figure 3. Sketch map of Combined Orbit Determination
**Measurement error separation**

Since the linearization of observation depends on the standard orbit, there exists the truncation error actually. In addition, the perturbation error of dynamical model would also be transferred to the observation equation, and there are also systematic error and random error in the observed data. General linearization method put these to the random error and it will influence the orbit estimate precision.
The method of observation error separation

Similar to the analysis of dynamical model perturbation error, we can separate the observation error by using sparse analysis. By analyze the O-C residual of BPS; we define the fitting basis as

$$\Delta r(t) = \sum_{i=0}^{M} a_i t^i + \sum_{k=1}^{N} (b_k \cos(\omega_k t) + c_k \sin(\omega_k t))$$

Where $\tilde{a} = (a_0, \ldots, a_M, b_1, \ldots, b_N, c_1, \ldots, c_N)^T$ represents the parameters to estimate, $\omega_k$ is the frequency to estimate, Here the maximum order $M, N$ of overcomplete basis can be determined by AIC principle.
result

Fig. 4 simulation of O-C residual

Fig. 5 The difference between simulation O-C residual and function fitting residual
3.4 estimation algorithm based on semiparametric regression model

- For the given observation series \( y_1, y_2, \ldots, y_m \), estimate the initial orbit \( x_0 \) by least square method;
- Compute the residual \( y'_1, y'_2, \ldots, y'_m \);
- Test \( y'_1, y'_2, \ldots, y'_m \) and see whether it is a white-noise series. If yes go to step5, else to step4;
Determine the function $\Delta r(t)$ by nonlinear fitting method and $\Delta r(t)$ satisfies:

$$
\begin{align*}
  y'_1 &= \Delta r(t_1) + \epsilon_1 \\
  y'_2 &= \Delta r(t_2) + \epsilon_2 \\
  &\cdots \\
  y'_m &= \Delta r(t_m) + \epsilon_m
\end{align*}
$$

Where $\epsilon_1, \epsilon_2, \cdots, \epsilon_m$ is a white-noise series, and replace $y_1, y_2, \cdots, y_m$ with $y_1 - \Delta r(t_1), y_2 - \Delta r(t_2), \cdots, y_m - \Delta r(t_m)$ and go to Step 1;

The output $x_0$ is the parameter to estimate.
4. Simulation

Fig.6 The user orbit determination results of position vector by the two algorithms respectively.
Table 1: The result comparison between the two orbit determination methods

<table>
<thead>
<tr>
<th>Different algorithm</th>
<th>Position error (m)</th>
<th>Velocity error (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>LS</td>
<td>24.0048</td>
<td>30.3501</td>
</tr>
<tr>
<td>Semiparametric</td>
<td>8.3980</td>
<td>5.6271</td>
</tr>
</tbody>
</table>
5. Conclusion

we constructed an orbit perturbation model fitted by overcomplete basis with an iterative way. The improved ESPRIT can separate the nonlinear and complex factors in dynamical model error and ensure the integrity of dynamical model by mathematical means. It can be added to the dynamical model to improve the model precision and simplify the solution of state equation. With the combination of the semiparametric algorithm, the nonlinear factors can be eliminated and the error of observation data can also be reduced. Therefore, our method outperforms the LS method a lot.
Thanks for your attention!!