Comparison of Particle Filters for Positioning

Duane Petrovich & Robert Piché
Tampere University of Technology, Finland
How to locate mobile devices using signals from satellites and cellular networks?

• Mathematical Formulation

  motion model: \[ x_k = f_{k-1}(x_{k-1}) + v_{k-1} \]

  measurement model: \[ y_k = h_k(x_k) + w_k \]
How to locate mobile devices using signals from satellites and cellular networks?

- **Mathematical Formulation**

  motion model: \( x_k = f_{k-1}(x_{k-1}) + v_{k-1} \)

  measurement model: \( y_k = h_k(x_k) + w_k \)

- **Recursive Bayesian Filter**

  \[
  p(x_k | y_{1:k-1}) = \int p(x_k | x_{k-1}) p(x_{k-1} | y_{1:k-1}) \, dx_{k-1}
  \]

  \[
  p(x_k | y_{1:k}) = \frac{p(y_k | x_k) p(x_k | y_{1:k-1})}{\int p(y_k | x_k) p(x_k | y_{1:k-1}) \, dx_k}
  \]
There are many ways to approximate posterior distribution
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How to compare the full distribution?

Position Estimate = Mean of Posterior
Use two-sample chi-squared test statistic to compare distributions

Linear-Gaussian Scenario,
\( \sigma_{\text{meas}} = 1 \text{ m}, \) time step \( k = 180 \)

\[
\chi^2 \\
\text{mean } \pm \text{ std } = 63 \pm 11.2
\]

\[
\text{mean } \pm \text{ std } = 276 \pm 50.6
\]
Use two-sample chi-squared test statistic to compare distributions

Linear-Gaussian Scenario, 
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\chi^2 \\
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\]

\[
\text{mean} \pm \text{std} = 276 \pm 50.6
\]

\[
\text{mean} \pm \text{std} = 66 \pm 11.75
\]
Use adaptive-binning for efficiency

The test is used to compare nonlinear filters

Nonlinear Scenario, range measurements, 3 base stations

500 m
The test is used to compare nonlinear filters.

Nonlinear Scenario, range measurements, 3 base stations.

Statistic mean ± std

<table>
<thead>
<tr>
<th>$\sigma_{\text{meas}}$</th>
<th>$\chi^2$</th>
<th>SIR1</th>
<th>SIR4</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 m</td>
<td>63 ± 11.2</td>
<td>1936 ± 1239</td>
<td>1763 ± 1951</td>
</tr>
<tr>
<td>5 m</td>
<td>63 ± 11.2</td>
<td>4429 ± 1190</td>
<td>2879 ± 737</td>
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<tr>
<td>1 m</td>
<td>63 ± 11.2</td>
<td>4016 ± 2439</td>
<td>1165 ± 677</td>
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