GNSS ambiguity resolution: when and how to fix or not to fix

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GNSS ambiguity resolution

GNSS Model: \( y \sim N(Aa + Bb, Q_y), \quad a \in Z^n, b \in R^p \)

Float solution: 
\[
\begin{bmatrix}
\hat{a} \\
\hat{b}
\end{bmatrix} \sim N(
\begin{bmatrix}
a \\
b
\end{bmatrix},
\begin{bmatrix}
Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\
Q_{\hat{b}\hat{a}} & Q_{\hat{b}}
\end{bmatrix}
)
\]

Integer estimation: \( \hat{a} \in R^n \Rightarrow \tilde{a} \in Z^n \)

Fixed solution: 
\[
\tilde{b} = \hat{b} - Q_{\hat{b}\tilde{a}}Q_{\tilde{a}}^{-1}(\hat{a} - \tilde{a})
\]

\( Q_{\tilde{b}} \ll Q_{\hat{b}} \) provided \( P(\tilde{a} = a) \approx 1 \)
Integer Estimation (Model-driven)

Choose integer map: \( \hat{a} \in R^n \Rightarrow \bar{a} \in Z^n \)

Evaluate failure rate: \( P_f = P(\bar{a} \neq a) = 1 - P(\bar{a} = a) \)

Decision:
\[
\begin{align*}
\text{If} \quad & P_f \leq \varepsilon \quad \text{use } \bar{a} \\
& P_f > \varepsilon \quad \text{use } \hat{a}
\end{align*}
\]

Remarks:
- Simple and valid approach: \( P_f \) known a priori => fixed/float known a priori
- Overall and rigorous quality description of \( \bar{a} \& \tilde{b} \) is available
- Optimal integer estimator = integer least-squares
- Model-based procedure: sample \( \hat{a} \) has no influence on decision
- User has no control over failure rate (other than strengthening the model)
Choice of integer estimators, e.g.

PMF: \[ P(\tilde{a} = z) = P(\hat{a} \in S_z) = \int_{S_z} f_\hat{a}(x) dx, \forall z \in \mathbb{Z}^n \]

Failure rate:
\[ P_f = 1 - P(\tilde{a} = a) \]
In practice the ‘Ratio Test’ is often used

Decision:

\[
\begin{align*}
\text{If } & \frac{\|\hat{a} - \tilde{a}\|_{Q,\hat{a}}^2}{\|\hat{a} - \tilde{a}\|_{Q,\hat{a}}^2} \leq \delta & \text{ use } \tilde{a} \\
\text{instead of } & \quad \text{If } P_f \begin{cases} 
\leq \varepsilon & \text{ use } \tilde{a} \\
> \varepsilon & \text{ use } \hat{a}
\end{cases}
\end{align*}
\]

Remarks:

• What role is played by the ‘ratio test’?
• Common motivation: it is a validation test (true/false?)
• With ‘ratio-test’ included, model-driven \( P_f \) is not applicable
• What is overall quality when ‘ratio test’ included?
• Current ways of choosing \( \delta \) are ad hoc or based on false theoretical grounds
Integer Aperture Estimation (Data-driven)

Integer aperture pull-in regions $\Omega_z$:  

\[
\bar{a} = \begin{cases} 
 z & \text{if } \hat{a} \in \Omega_z \\
 \hat{a} & \text{if } \hat{a} \notin \bigcup_z \Omega_z 
\end{cases}
\]

Success rate:  
\[P_s = P(\hat{a} \in \Omega_a)\]

Failure rate:  
\[P_f = \sum_{z \neq a} P(\hat{a} \in \Omega_z)\]

Undecided rate:  
\[P_u = 1 - \sum_z P(\hat{a} \in \Omega_z)\]
Various Integer Aperture Estimators Possible

Choose failure rate $P_f$ => sets aperture of $\Omega_z$

Remarks:
• User has control over failure rate
• Data-driven: sample $\hat{a}$ has influence on decision
• Success-rate and failure-rate can be computed
‘Ratio Test’ revisited

‘Ratio Test’ = Integer Aperture Estimation

\[
\frac{\|\hat{a} - \bar{a}\|^2_{Q_{\hat{a}}}}{\|\hat{a} - \bar{\bar{a}}\|^2_{Q_{\hat{a}}}} \leq \delta \iff \hat{a} \in \Omega \quad \text{with} \quad \Omega = \bigcup_{z \in \mathbb{Z}^n} \Omega_z, \quad \Omega_0 + z = \Omega_z
\]

\[
\Omega_0 = \left\{ x \in \mathbb{R}^n \left\| x \right\|_{Q_{\hat{a}}}^2 \leq \delta \left\| x - z \right\|_{Q_{\hat{a}}}^2, \forall z \in \mathbb{Z}^n \setminus \{0\}, \ 0 \leq \delta \leq 1 \right\}
\]

‘Ratio Test’ Aperture Pull-in regions:
‘Ratio test’ revisited

• ‘Ratio test’ = Integer Aperture Estimator
• ‘Ratio test’ is not a validation test
• User can have control over failure rate
• Choice of δ should be coupled to choice of acceptable failure rate \( \rightarrow \) this is not current practice
On the choice of \( \delta \)

6 dual-frequency GPS models
Optimal Integer Aperture estimation

Aperture pull-in region $\Omega_z$ chosen such that success rate is maximized for a fixed failure rate:

$$\max_{\Omega_0} P_s \quad \text{subject to} \quad P_f = \sum_{z \neq a} \int_{\Omega_z} f_{\hat{a}}(x) dx = \beta$$

Solution:

$$\Omega_0 = \left\{ x \in S_0 \left| \sum_{z \in Z^n} f_{\hat{a}}(x + z) \leq \mu f_{\hat{a}}(x + a) \right. \right\}$$

size controlled by $\mu$

with $S_0$ the integer least-squares pull-in region
Optimal Integer Aperture estimation

\[ \Omega_0 = \left\{ x \in S_0 \left| \sum_{z \in \mathbb{Z}^n} f_{\hat{a}}(x + z) \leq \mu f_{\hat{a}}(x + \hat{a}) \right. \right\} \]

Computational steps:
1. Given \( \hat{a} \) determine integer least-squares estimate \( \tilde{a} \)
2. Compute ambiguity residual \( \tilde{\epsilon} = \hat{a} - \tilde{a} \)
3. \[ \begin{cases} \tilde{\epsilon} \in \Omega_0 & \text{use } \tilde{a} \\ \tilde{\epsilon} \notin \Omega_0 & \text{use } \hat{a} \end{cases} \]
Conclusions

• User controllable failure rate: Integer aperture estimation

• ‘Ratio Test’ = Integer aperture estimator

• $\delta$ should be computed from chosen failure rate

• ‘Ratio Test’ $\neq$ Optimal integer aperture estimator

• practical relevance: time to first fix shorter and probability of incorrect fixing below threshold

• efficient implementation by LAMBDA
http://www.lr.tudelft.nl/mgp
Ratio Test vs. Optimal IA estimator

\[ Q_1 = \begin{pmatrix} 0.0577 & -0.0242 \\ -0.0242 & 0.0564 \end{pmatrix}; \quad Q_2 = 6Q_1 \]

blue: Optimal test
red: Ratio Test
Ratio Test vs. Optimal IA estimator

The plots show the relationship between fail rate and percentage identical for different conditions labeled as 'a', 'b', and 'c'. The graphs illustrate the performance of the Ratio Test and the Optimal IA estimator across varying fail rates.
Ratio Test vs. Optimal IA estimator

$P_f = 0.005$