Do we need new gravity field recovery techniques for the new gravity field satellites?

Karl Heinz Ilk
Institute of Theoretical Geodesy
University Bonn

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Challenge and Role of Modern Geodesy
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Contents

- The classical analysis techniques of dynamic satellite geodesy
- The generation of new satellite missions: densely observed orbits and high-precise sensors
- The in-situ measurement procedure and tailored analysis techniques
- A new scheme of integrals of motion for validation and gravity field improvement
- Do we face a new era of gravity field analysis techniques?
The classical techniques of Satellite Geodesy (1965-2000)
### Physical models of Satellite Geodesy

#### Newton-Euler-Formalism

\[ m \frac{d}{dt} \mathbf{p}(t) = \mathbf{K}(\mathbf{r}, \dot{\mathbf{r}}; t) \]

- Force function: \( \mathbf{K}(\mathbf{r}, \dot{\mathbf{r}}; t) \)
- Cartesian coordinates: \( \mathbf{r} \)
- Linear momentum: \( \mathbf{p} \)

#### Lagrange-Formalism

\[ \frac{d}{dt} \nabla_q L(q, \dot{q}; t) = \nabla_q L(q, \dot{q}; t) \]

- Lagrange function: \( L(q, \dot{q}; t) \)
- Generalized coordinates: \( q \)
- Generalized velocities: \( \dot{q} \)

#### Hamilton-Formalism

\[ \dot{q} = \nabla_p H(p, q; t) \]
\[ \dot{p} = \nabla_q H(p, q; t) \]

- Hamilton function: \( H(p, q; t) \)
- Generalized coordinates: \( q \)
- Generalized momentum: \( p \)

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### Numerical and analytical solutions

- **Numerical solutions:** numerical perturbation theories
- **Analytical solutions:** analytical perturbation theories
Modelling: Numerical and analytical orbit perturbation techniques
Explicite Lagrange’s perturbation equations

\[
\frac{di}{dt} = \frac{1}{na^2 \sqrt{1-e^2} \sin i} \left( \cos i \frac{\partial R}{\partial \omega} - \frac{\partial R}{\partial \Omega} \right)
\]

\[
\frac{d\Omega}{dt} = \frac{1}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial R}{\partial i}
\]

\[
\frac{d\omega}{dt} = \frac{1}{na^2 \sqrt{1-e^2} \sin i} \left( -\cos i \frac{\partial R}{\partial i} + \frac{1-e^2}{e} \sin i \frac{\partial R}{\partial e} \right)
\]

\[
\frac{da}{dt} = \frac{2}{na} \frac{\partial R}{\partial \sigma}
\]

\[
\frac{de}{dt} = \frac{1}{na^2 e} \left( -\sqrt{1-e^2} \frac{\partial R}{\partial \omega} + (1-e^2) \frac{\partial R}{\partial \sigma} \right)
\]

\[
\frac{d\sigma}{dt} = \frac{1}{na^2 e} \left( -2ae \frac{\partial R}{\partial a} - (1-e^2) \frac{\partial R}{\partial e} \right)
\]
Explicite Lagrange’s perturbation equations (Kaula’s expansion)

\[ \frac{di}{dt} = \sum_{n,m,p,q} GM_\odot a^n \frac{F_{nmpl} G_{pq} S'_{nmpl}}{\sqrt{GM_\odot a(1-e^2)}a^{n+1} \sin i} (n-2p) \cos i - m \]

\[ \frac{d\Omega}{dt} = \sum_{n,m,p,q} GM_\odot a^n \frac{\partial F_{nmpl} / \partial G_{pq} S_{nmpl}}{\sqrt{GM_\odot a(1-e^2)}a^{n+1} \sin i} \]

\[ \frac{d\omega}{dt} = \sum_{n,m,p,q} GM_\odot a^n \frac{S_{nmpl}}{\sqrt{GM_\odot a} a^{n+1}} \left( \frac{\sqrt{1-e^2}}{e} F_{nmpl} \frac{\partial G_{pq}}{\partial e} - \cot i \frac{\partial F_{nmpl}}{\partial e} G_{pq} \right), \]

\[ \frac{da}{dt} = \sum_{n,m,p,q} GM_\odot a^n \frac{\sqrt{a}}{\sqrt{GM_\odot}} F_{nmpl} G_{pq} S'_{nmpl} \frac{n-2p+q}{a^{n+1}} \]

\[ \frac{de}{dt} = \sum_{n,m,p,q} GM_\odot a^n F_{nmpl} G_{pq} S'_{nmpl} \left( \frac{\sqrt{1-e^2} (n-2p+q)-(n-2p)}{\sqrt{GM_\odot a^{n+1}}} \right) \frac{1-e^2}{e} \frac{\partial G_{pq}}{\partial e} \]

\[ \frac{dM}{dt} = \sqrt{\frac{GM_\odot}{a^3}} + \sum_{n,m,p,q} GM_\odot a^n \frac{\sqrt{a}}{\sqrt{GM_\odot} a^{n+1}} F_{nmpl} S_{nmpl} \left( 2(n+1)G_{pq} - \frac{1-e^2}{e} \frac{\partial G_{pq}}{\partial e} \right). \]
## Analytical solutions to the perturbation equations

<table>
<thead>
<tr>
<th>index combination filled</th>
<th>type of disturbances</th>
<th>disturbance possible for element</th>
<th>remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ $m$ $n-2p$ $n-2p+q$ $q$ $N = \frac{\pi}{\dot{\Omega}}$</td>
<td>secular</td>
<td>periodic with period</td>
<td></td>
</tr>
<tr>
<td>even 0 0 0 0</td>
<td>arbitrary</td>
<td>×</td>
<td>$\Omega, \omega, M$</td>
</tr>
<tr>
<td>0 ≠ 0 ≠ 0 ≠ 0</td>
<td>$\frac{2\pi}{(n-2p+q)\dot{\pi}}$</td>
<td>$i, \Omega, \omega, a, e, M$</td>
<td>pronounced for $</td>
</tr>
<tr>
<td>0 ≠ 0 0 0</td>
<td>$\frac{2\pi}{(n-2p)\dot{\omega}}$</td>
<td>$i, \Omega, \omega, e, M$</td>
<td>occurs only for $n$ odd, if in $R$ also a even $n$ present ist; long periodic (apside revolution) (especially for $</td>
</tr>
<tr>
<td>≠ 0</td>
<td>discrete</td>
<td>×</td>
<td>only, if $\pi$ and $\dot{\Theta}$ rigorously commensurable</td>
</tr>
<tr>
<td>≠ 0 0</td>
<td></td>
<td></td>
<td>periodic (sidereal day)</td>
</tr>
<tr>
<td>≠ 0 ≠ 0</td>
<td></td>
<td></td>
<td>periodic resonances possible</td>
</tr>
<tr>
<td>≠ 0 ≠ 0 0</td>
<td></td>
<td></td>
<td>long periodic</td>
</tr>
<tr>
<td>0 0</td>
<td>no disturbance</td>
<td></td>
<td>$i$</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>$a$</td>
</tr>
<tr>
<td>0 0 0</td>
<td></td>
<td></td>
<td>$e$</td>
</tr>
</tbody>
</table>
Definitive orbit determination by differential corrections

**basic geometric relation**
\[
\mathbf{r}_i(t) = \mathbf{R}_{li}(t) + \mathbf{R}_i(t)
\]

**orbit model (equation of motion)**
\[
\dot{\mathbf{r}}_i(t) = \mathbf{a}_K(t) + \mathbf{a}_R(t; \mathbf{x}_R) + \mathbf{a}_{St}(t; \mathbf{x}_i)
\]

**numerical integration**

**non linear mathematical/physical model**
\[
\mathbf{r}_i(t_k; \alpha_i^0 + d\alpha_i, \mathbf{x}_i^0 + d\mathbf{x}_i, \mathbf{x}_R^0 + d\mathbf{x}_R) = \mathbf{r}_i(t_k; \overline{\mathbf{b}}_i + d\mathbf{b}_i; \mathbf{x}_S^0 + d\mathbf{x}_S)
\]

**observation model**
\[
\mathbf{r}_i(t_k) = \mathbf{R}_{li}(t_k; \overline{\mathbf{b}}_i) + \mathbf{R}_i(t_k; \mathbf{x}_S^0)
\]

**linearized mathematical/physical model**
\[
\left( \frac{D\mathbf{r}_i(t_i)}{D\alpha_i} \right) \cdots 0 \left( \frac{D\mathbf{r}_i(t_i)}{D\mathbf{x}_i} \right) \left( \frac{D\mathbf{r}_i(t_i)}{D\mathbf{x}_R} \right) - \left( \frac{D\mathbf{r}_i(t_i)}{D\mathbf{x}_S} \right) \left( \frac{d\alpha_i}{\cdots d\alpha_L} \right) \left( \frac{d\mathbf{x}_i}{\cdots d\mathbf{x}_R} \right) \left( \frac{d\mathbf{x}_S}{\cdots d\mathbf{x}_S} \right)
\]

\[
= \left( \mathbf{r}_i(t_i; \mathbf{x}_S^0, \overline{\mathbf{b}}) - \mathbf{r}_i(t_i; \alpha_i^0, \mathbf{x}_i^0, \mathbf{x}_R^0) \right) + \left( \frac{D\mathbf{r}_i(t_i)}{D\overline{\mathbf{b}}_i} \right) 0 0 \left( \frac{d\mathbf{b}_i}{\cdots d\mathbf{b}_L} \right)
\]
A new generation of satellites: Satellite-to-Satellite Tracking (SST) and Satellite-Gravitations-Gradiometrie (SGG)

GOCE

GOCE gradiometer

CHAMP

GRACE
The measurement of the tidal field by Satellite-to-Satellite tracking (SST)

\[ \mu_{12} \ddot{R}_{12} = K_{21} + G_{(21)3} \]
The measurement of the tidal field by Satellite-Gravity-Gradiometry (SGG)

\[ K_{12} = e_{12} \cdot G_{(21)3} (R_{12}, R_1) = m \frac{R_{12}}{4} \left( V_{yy} + V_{zz} + \cos(2\theta)(V_{zz} - V_{yy}) + 2\sin(2\theta)V_{yz} \right). \]

\[ K_{12} = \frac{m}{2} R_{12} V_{zz} \]
<table>
<thead>
<tr>
<th>Classical techniques</th>
<th>New techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td>few observations, most of them laser ranging, use of long arcs with moderate accuracy, analysis of gravity field disturbances, accumulated over a long observation period.</td>
<td>orbits with dense observation coverage, enabled by precise satellite positioning systems, use of short arcs possible. Possibility of in-situ measurements.</td>
</tr>
<tr>
<td>Application of differential orbit improvement techniques, analytical perturbation techniques.</td>
<td>In-situ analysis techniques by direct measurements of gravity field functionals, application of general integrals of motion.</td>
</tr>
<tr>
<td>Accumulation of unmodelled disturbance forces, use of base functions with global support (spherical harmonics), use of different satellites with varying inclinations, long accumulation periods do not allow to derive temporal variations of the gravity field.</td>
<td>Accumulation of disturbance forces can be avoided, use of base functions with global and local support, global and regional gravity field determinations, one satellite with high inclination is sufficient, temporal variations of the gravity field can be derived because of shorter analysis periods.</td>
</tr>
</tbody>
</table>
In-situ gravity field determination based on densely observed satellite orbits

1. \( \mathbf{r}_i(t) \)
2. \( \mathbf{r}_i'(t) \)
3. \( \mathbf{r}_i''(t) \)

Integral equation of Fredholm type
\[ \mathbf{r}(t) = \mathbf{r}(t) \left( 1 - \frac{1}{m} \int_{t_0}^{t} K(t, t') \mathbf{K}(\mathbf{r}, \mathbf{r}; t') \, dt' \right) \]

Integrals of motion
\[ \mathbf{r}'(t) = \frac{1}{m} \int_{t_0}^{t} K(\mathbf{r}, \mathbf{r}; t) \, dt \]

Equation of motion
\[ \mathbf{r}'(t) = \frac{1}{m} \mathbf{K}(\mathbf{r}, \mathbf{r}; t) \]

Solved for:
- twofold integration
- simple integration
- observed

Simple differentiation
- twofold differentiation
Integral equation of Fredholm type

\[ r(t) = \overline{r}(t) - \frac{1}{m} \int_{t_0}^{t} K(t, t') K(r, \dot{r}; t) \, dt \]

Analysis in space domain (Mayer-Gürr et al. Uni Bonn)

\[ r(t) = \overline{r}(t) - \frac{1}{m} \int_{t_0}^{t} K(t, t') K(r, \dot{r}; t) \, dt \]

Analysis in spectral domain (Ilk et al. Uni Bonn)

\[ r(t) = \overline{r}(t) + \sum_{\nu=1}^{\infty} r_{\nu} \sin(\nu \pi t) \]

\[ r_{\nu} = -\frac{2T^2}{m \pi^2 \nu^2} \int_{t' = 0}^{1} \sin(\nu \pi t') K(r, \dot{r}; t) \, dt' \]
Analysis level 3

**Equation of motion**

\[ \ddot{r}(t) = \frac{1}{m} K(r, \dot{r}; t) \]

- Weighted averages (Ditmar et al. (DEOS Delft))
- Moving interpolation (Reubelt et al. (Uni Stuttgart))
Analysis level 2: classical balance equations

**Integrals of motion**

**Linear momentum**
\[ M\ddot{\mathbf{R}} - \mathbf{K} = 0 \]

**Angular momentum**
\[ M\ddot{\mathbf{R}} - \mathbf{K} = 0 \]

*Vector multiplication with \( \dot{R} \)*
\[ M\mathbf{R} \times \dot{\mathbf{R}} - \mathbf{R} \times \mathbf{K} = 0 \]

**Energy balance**
\[ M\ddot{\mathbf{R}} - \mathbf{K} = 0 \]

*Scalar multiplication with \( \dot{R} \)*
\[ M\mathbf{R} \cdot \dot{\mathbf{R}} - \mathbf{R} \cdot \mathbf{K} = 0 \]

**timewise integration**
\[ M\dot{\mathbf{R}} - \int_{t_0}^{t} \mathbf{K} \, dt = \mathbf{P}_0 \]

Linear momentum = initial value + integrated force

**timewise integration**
\[ M\mathbf{R} \times \dot{\mathbf{R}} - \int_{t_0}^{t} \mathbf{R} \times \mathbf{K} \, dt = \mathbf{L}_0 \]

Angular momentum = initial value + integrated torque

**timewise integration**
\[ \frac{1}{2} M\dot{\mathbf{R}}^2 - \int_{t_0}^{t} \mathbf{K} \cdot \dot{\mathbf{R}} \, dt = E \]

Energy = kinetic energy - Integrated work
Energy balance in a Terrestrial Reference Frame

\[ M\ddot{R}' - Z' - C' - K' = 0 \]

**Zentrifugal force**

\[ Z' = -M (\Omega' \times (\Omega' \times R')) \]

**Coriolis force**

\[ C' = -2M (\Omega' \times \dot{R}') \]

\[ M\ddot{R}' + M (\Omega' \times (\Omega' \times R')) + 2M (\Omega' \times \dot{R}') - K' = 0 \]

**Scalar multiplication with \( \dot{R} \)**

\[ M\ddot{R}' \cdot \dot{R}' - M (\Omega' \times R') \cdot (\Omega' \times \dot{R}') - K' \cdot \dot{R}' = 0 \]

**Timewise integration**

\[ \dot{E}' = \frac{1}{2} M\dot{R}'^2 - \frac{1}{2} M (\Omega' \times R')^2 - MV' + \dot{E}'_0 \]

**Jacobi-Integral**

\( \Omega' \): angular velocity of the Earth (considered as constant)

potential
Formulating balance equations

\[ M \ddot{\mathbf{R}} - \mathbf{K} = 0 \]

Transformation of Newton-Euler-equation

\[ f(M, \mathbf{R}, \dot{\mathbf{R}}, \ddot{\mathbf{R}}) - g(\mathbf{R}, \dot{\mathbf{R}}, \mathbf{K}) = 0 \]

timewise integration

\[ F(M, \mathbf{R}, \dot{\mathbf{R}}) - \int_{t_s}^{t} g(\mathbf{R}, \dot{\mathbf{R}}, \mathbf{K}) dt = C \]

„kinetic“ term

force integral
Integrals of translational motion

\[ \mathbf{M} \dot{\mathbf{R}} = \mathbf{K} = 0 \]

- Considered as a vector equation
- Time integration
- Scalar multiplication by \( \mathbf{R} \)

\[ \int_{t_0}^{t} \mathbf{K} \cdot \mathbf{R} \, dt = \mathbf{P}_0 \]

b. e. of the linear momentum \( \text{kg m/s} \)

\[ \frac{1}{2} \mathbf{M} \dot{\mathbf{R}}^2 - \int_{t_0}^{t} \mathbf{K} \cdot \mathbf{R} \, dt = E \]

total energy \( \text{kg m}^2/\text{s}^2 \)

\[ \frac{1}{2} \mathbf{M} \dot{\mathbf{R}}^2 - \int_{t_0}^{t} \mathbf{K}_i \, dt = E_i \]

energy in the coordinates \( \text{kg m}^2/\text{s}^2 \)

\[ \mathbf{M} \dot{\mathbf{R}}_i - \int_{t_0}^{t} (\mathbf{K}_i \dot{\mathbf{R}}_i + \mathbf{K}_i \dot{\mathbf{R}}_i) \, dt = E_i \]

energy in the coordinate surfaces \( \text{kg m}^2/\text{s}^2 \)

\[ \mathbf{M} \dot{\mathbf{R}}_i \mathbf{R}_i - \int_{t_0}^{t} (\mathbf{K}_i \dot{\mathbf{R}}_i \mathbf{R}_i + \mathbf{K}_i \dot{\mathbf{R}}_i \mathbf{R}_i + \mathbf{K}_i \dot{\mathbf{R}}_i \mathbf{R}_i) \, dt = E_{ii} \]

b. e. of the momentum volume \( \text{kg m}^3/\text{s}^3 \)
Integrals of translational motion

\[ M \dot{\mathbf{R}} - \mathbf{K} = 0 \]

considered as a vector equation

time integration

\[ \frac{1}{2} M \dot{\mathbf{R}}^2 - \int t \mathbf{K} \cdot \dot{\mathbf{R}} dt = E \]

total energy \( \text{kg} \, m^2 / s^2 \)

\[ \frac{1}{2} M \dot{\mathbf{R}}^2 - \int t \mathbf{K}_i \cdot \dot{R}_i dt = E_i \]

energy in the coordinates \( \text{kg} \, m^2 / s^2 \)

\[ \frac{\partial E}{\partial \mathbf{R}} = \mathbf{P} \]

\[ E_{xx} + E_{yy} + E_{zz} = E \]

\[ \frac{\partial E_{xx}}{\partial \mathbf{R}_i} = P_i \]

\[ \frac{\partial E_{yy}}{\partial \mathbf{R}_j} = P_j \]

\[ \frac{\partial E_{zz}}{\partial \mathbf{R}_k} = P_k \]

b. e. of the linear momentum \( \text{kg} \, m / s \)

time integration

\[ M \dot{\mathbf{R}}_i - K_i = 0 \]

\( i \in \{x, y, z\} \)

considered as a triple of scalar equations

multiplying two equations crosswise by \( \dot{R}_i \) and \( R_j \)

adding the pairs of equations

multiplying each equation by \( \dot{R}_i \), \( \dot{R}_j \)

adding the triple of equations

multiplying each equation by the other components of the velocity

integrals of translational motion
Integrals of rotational motion

\[ M\mathbf{\dot{R}} - \mathbf{K} = 0 \]

\[ M\mathbf{\dot{R}} \times \mathbf{R} - \mathbf{M} = 0 \]

\[ \mathbf{L} - \int_{t_i}^{t_f} \mathbf{M} \cdot d\mathbf{R} = \mathbf{L}_0 \]

\[ \frac{1}{2} M \mathbf{L}^2 - \frac{1}{M} \int_{t_i}^{t_f} \mathbf{M} \cdot \mathbf{L} \, dt = \mathcal{E} \]

\[ \frac{1}{2} M \mathbf{L}_i^2 - \frac{1}{M} \int_{t_i}^{t_f} \mathbf{M} \cdot \mathbf{L}_i \, dt = \mathcal{E}_i \]

\[ \frac{1}{M} \mathbf{L}_i \mathbf{L}_j - \frac{1}{M} \int_{t_i}^{t_f} (\mathbf{M} \mathbf{L}_i + \mathbf{M} \mathbf{L}_j) \, dt = \mathcal{E}_ij \]

\[ \frac{1}{M} \mathbf{L}_i \mathbf{L}_j \mathbf{L}_k - \frac{1}{M} \int_{t_i}^{t_f} (\mathbf{M} \mathbf{L}_i \mathbf{L}_j + \mathbf{M} \mathbf{L}_j \mathbf{L}_k + \mathbf{M} \mathbf{L}_k \mathbf{L}_i) \, dt = \mathcal{E}_ijk \]

\[ i \in \{x, y, z\} \]

b. e. of the angular momentum vector \( \mathbf{R} \)

b. e. of the angular momentum volume \( \mathbf{R} \times \mathbf{R} \)

Vector multiplication by \( \mathbf{R} \)

Scalar multiplication by \( \mathbf{R} \times \mathbf{R} \)

Multiplying each equation by \( (\mathbf{R} \times \mathbf{R}) \)

Multiplying two equations crosswise by \( (\mathbf{R} \times \mathbf{R}) \) and \( (\mathbf{R} \times \mathbf{R}) \)

Multiplying each equation by the other components of \( \mathbf{R} \times \mathbf{R} \)

Time integration

Time integration

Time integration

Time integration

Time integration

Time integration

Time integration

Time integration

Time integration

Total rotational energy \( \text{kg m}^2 / \text{s} \)

Rotational energy in the coordinates \( \text{kg m}^2 / \text{s}^2 \)

Rotational energy in the coordinate surfaces \( \text{kg m}^4 / \text{s}^2 \)

B. E. of the angular momentum volume \( \text{kg m}^4 / \text{s}^3 \)
Integrals of rotational motion

\[ M\overset{\cdot}{\mathbf{R}} - \mathbf{K} = 0 \]

Considered as a vector equation

\[ \int_{t_0}^{t} \mathbf{M} \, dt = \mathbf{L}_0 \]

b. e. of the angular momentum \( \text{kg} \, m^2 / s \)

\[ \frac{1}{2M} \mathbf{L}^2 - \frac{1}{M} \int_{t_0}^{t} \mathbf{M} \cdot \mathbf{L} \, dt = \mathcal{E} \]

Total rotational energy \( \text{kg} \, m^4 / s^2 \)

\[ \frac{\partial \mathcal{E}}{\partial (\mathbf{R} \times \mathbf{R})} = \mathbf{L} \]

\[ \mathcal{E}_x + \mathcal{E}_y + \mathcal{E}_z = \mathcal{E} \]

\[ \frac{1}{M} L_i L_j - \frac{1}{M} \int_{t_0}^{t} (M L_i + M_i L_j) \, dt = \mathcal{E}_{ij} \]

Rotational energy in the coordinates \( \text{kg} \, m^4 / s^2 \)

\[ \frac{\partial \mathcal{E}_{ij}}{\partial (\mathbf{R} \times \mathbf{R})} = L_i \]

\[ \int_{t_0}^{t} (M L_i + M_i L_j) \, dt = \mathcal{E}_{ij} \]

\[ \frac{1}{M} L_i L_j \mathcal{L}_k - \frac{1}{M} \int_{t_0}^{t} (M L_i L_j + M_i L_j L_k + M_j L_i L_k) \, dt = \mathcal{E}_{ijk} \]

b. e. of the angular momentum volume \( \text{kg} \, m^6 / s^3 \)
Numerical tests

per integral of motion two parameter estimations:
- without rigorous variance-covariance propagation
- with rigorous variance-covariance propagation

Data: 30 days kinematic CHAMP orbits + accelerometer data

Reference („true“) gravity field model: EIGEN_GRACE1S

Estimated parameters:
- potential coefficients up to degree 70
- per orbit 3 accelerometer biases
- per arc 1 and 3 integration constants, respectively

Problem: with error propagation bad condition of the system of normal equations
Way out: short arcs (15 minutes)

Problem: covariance matrices of the integrals have partly large rank defects
   (caused by differentiation and vector multiplication)
Way out: determination of the weight matrices by pseudo inverses
Results without rigorous variance-covariance propagation

- Formal error of $c_{nm}$
- "True" error reference: EIGEN-GRACE1S

Dimensions: 595.2x842.0
Results with rigorous variance-covariance propagation

- **Formal error of $c_{nm}$**
- **"True" error reference: EIGEN GRACE1S**

1. **Linear Momentum**
2. **Total Energy**
3. **Energy Coordinates**
4. **Energy Coordinate Planes**
5. **Linear Momentum Volume**
6. **Angular Momentum**
7. **Rotational Energy**
8. **Rotational Energy Coordinates**
9. **Rotational Energy Planes**
10. **Angular Momentum Volume**
Linear momentum (without radial component), angular momentum and total energy
The orbit system

cross track component

along track component

radial component

osculating orbit plane
Linear momentum, with radial and without radial component

- Linear momentum
- Linear momentum without radial term
- Difference
Linear momentum, energy coordinate and energy coordinate plane projections

- Linear momentum
- Energy coordinates
- Energy coordinate planes
Angular momentum, rotational energy coordinate and rotational energy coordinate plane projections
Error degree variances: without rigorous variance-covariance propagation

True errors: differences to the gravity field model EIGEN_GRACE1S

Formal errors as a result of the adjustment
Error degree variances: with rigorous variance-covariance propagation

True errors: differences to the gravity field model EIGEN_GRACE1S

Formal errors as a result of the adjustment
Remarks to the integral of angular momentum

Effect of an error in the velocity on the angular momentum

\[
\delta L = MR \times \delta \vec{R} = MR \times (\delta \vec{R}_R + \delta \vec{R}_\perp) = MR \times \delta \vec{R}_\perp
\]

\(\delta \vec{R}_R\): velocity error in radial direction  
\(\delta \vec{R}_\perp\): perpendicular component

There is no sensitivity of the integral of linear momentum against velocity errors in radial direction.

Legend:
- integral of linear momentum
- integral of angular momentum without radial component
- integral of angular momentum
Remarks to the energy balances

Effect of an error in the velocity on the kinetic energy

\[ \delta E = M\ddot{R} \cdot \delta \ddot{R} \]
\[ = M\ddot{R} \cdot (\delta \dot{R}_R + \delta \dot{R}_\perp) \]
\[ = M\ddot{R} \cdot \delta \dot{R}_R \]

\( \delta \dot{R}_R \) : velocity error in along track direction
\( \delta \dot{R}_\perp \) : perpendicular component

There is no sensitivity of the kinetic energy against errors in the velocity, perpendicular to the velocity

Effect of an error in the velocity on the rotational energy

\[ \delta E_{rot} = M (R \times \dot{R}) \cdot (R \times \delta \dot{R}) \]
\[ = M ((R \times \dot{R}) \times R) \cdot \delta \dot{R} \]
\[ \approx M ((R \times \dot{R}) \times R) \cdot \delta \dot{R}_R \]

Circular orbits \( (R \times \dot{R}) \times R \times \dot{R} \)

\( \delta \dot{R}_R \) : see above

In case of circular orbits there is no sensitivity of the rotational energy against velocity errors, perpendicular to the along track direction

In case of circular orbits energy and rotational energy against give identical results
Do we face a new era of gravity field analysis techniques?

The classical analysis techniques which are used by the expert institutes in producing gravity field models are still in front of determining precise static and temporal gravity field models.

Nevertheless: Alternative analysis techniques, tailored to the characteristics of the new satellite missions show some attractive advantages

- Use of short arcs and in-situ measurement characteristics can minimize the influence of disturbing forces which tend to accumulate over longer arcs
- Flexible with respect to field modelling (base functions with global and local support)
- Flexible with respect to analysis area (global and regional gravity field improvement possible)
- Flexible with respect to a combination with terrestrial data
- Use for gravity field improvement and consistency validation of orbits and field functions
Thank you very much for your attention