Atmospheric De-Aliasing revisited

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Motivation

• **GRACE**: Determination of the Earth's gravity field and its temporal variations

• Atmospheric mass distribution required for:
  - De-Aliasing of short-term variations
  - Separation from other geofluids

• Errors in atmospheric modeling propagate to gravity field models and complicate geophysical interpretation

• **GOCE**: temporal variations are noise, corrections necessary
Outline

• Modeling atmospheric gravity variations

• Case studies with operational ECMWF data

• Conclusions
Modeling atmospheric mass

- Ecker & Mittermayer (1969): → static standard atmosphere


- Christodoulidis (1976, 1979): → dependency of lat, height & season

-...

- GRACE-AOD1B (Flechtner, 2005) → uses output of meteorological models
Modeling with ECMWF data

\[ \frac{\bar{C}_{nm}(t)}{\bar{S}_{nm}(t)} \equiv \frac{(1 + k'_n)}{(2n + 1)} \cdot \frac{3}{4\pi R \bar{\rho}} \int_{-\infty}^{+\infty} I_n(t) \bar{P}_{nm} \left\{ \begin{array}{l} \cos m \lambda \\sin m \lambda \end{array} \right\} d\sigma \]

with \( I_n(t) = \int_{0}^{+\infty} \left( \frac{r}{R} \right)^{n+2} \rho(t) \, dr \) and \( \rho(t) = \frac{p(t)}{R_d T_v(t)} \)

Hydrostatic equilibrium: \( dp(t) = -g \rho(t) \, dr \)

\[ I_n(t) = \frac{p_s(t)}{g} \quad I_n(t) = - \int_{p=p_s(t)}^{0} \left( \frac{r}{R} \right)^{n+2} \frac{R_d T_v(t)}{p(t) g} \, dp(t) \]

Surface pressure on sphere \quad Vertical integration on realistic earth

with gas constant dry air \( R_d \), virtual temperature \( T_v \)
Vertical Integration - ECMWF

Model provides surface pressure $p_s$, level-defining coefficients $A_{k+1/2}$, $B_{k+1/2}$, surface geopotential $\Phi_s$, temperature $T_k$ and specific humidity $q_k$ per level $k$

\[
p_{k+1/2}(t) = A_{k+1/2} + B_{k+1/2} \cdot p_s(t)
\]

\[
T_{v,k}(t) = T_k(t)(1 + 0.6078 \cdot q_k(t))
\]

\[
\Phi_{k+1/2}(t) = \Phi_s + \sum_{j=k+1}^{k_{max}} R_d T_{v,j}(t) \ln \left( \frac{p_{j+1/2}(t)}{p_{j-1/2}(t)} \right)
\]

Open issues:
- Relationship radial coordinate and geopotential $(r = ?)$
- Body used for computation $(R = ?)$
- Approximation for gravity $(g = ?)$

see Swenson and Wahr (2002) JGR; Boy and Chao (2005) JGR; Flechtner (2005) GRACE; ... for details
**Modeling questions**

\[ r(t) = R' + N + \Phi(t)/g_0 \]

- \( R' \): Ellipsoidal geocentric radius
- \( N \): Geoid height
- \( \Phi/g_0 \): Geopotential height

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Differences in ( I_n )</th>
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<tbody>
<tr>
<td>Different values for ( g_0 )</td>
<td>Scaling</td>
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<tr>
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<td>0.3 to 2 Pa</td>
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<td>( R + N ) vs. ( R' + N )</td>
<td>Systematic with lat</td>
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<td>1.5 to 20 Pa</td>
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<td>( g ) as normal gravity, exact conversion of geopotential height</td>
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<td>20 Pa to hPa</td>
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Effects reduced by subtraction of mean field!
Vertical integration and GOCE

RMS geoid amplitudes [m],
Mean values from 2 months 6h data

Mean difference expressed
in geoid height [mm]
ECMWF model updates

Model update from 60 to 91 levels on Feb 1, 2006

RMS geoid amplitudes [m],
Mean, min and max from 1 month 6h data
### ECMWF model updates (2)

<table>
<thead>
<tr>
<th>Year</th>
<th>$C_{00}$ in Geoid Height [m]</th>
<th>$C_{10}$ in Geoid Height [m]</th>
<th>$C_{11}$ in Geoid Height [m]</th>
<th>$S_{11}$ in Geoid Height [m]</th>
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**Chart:**

- $C_{00}$
- $C_{10}$
- $C_{11}$
- $S_{11}$

**Legend:**

- Black: $C_{00}$
- Red: $C_{10}$
- Blue: $C_{11}$
- Green: $S_{11}$
Mean atmospheric gravity


ΔN [mm]

-8 | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8
Accuracy of atmospheric models

ECMWF vs. NCEP - case surface pressure only

RMS geoid amplitudes of typical monthly mean [m]

Corresponding geoid heights [mm]
Accuracy of atmospheric models

ECMWF vs. NCEP - without difference in mean field

RMS geoid amplitudes [m]: Mean, min and max from 3 months 6 h data

Corresponding mean geoid heights [mm]
Conclusions

1. Careful modeling necessary of
   a) Earth's shape
   b) normal gravity
   c) conversion from geopotential to height
2. Vertical integration of atmospheric column is essential
3. Using operational data pay attention to
   a) model updates
   b) computation of mean atmospheric gravity
4. Accuracy of atmospheric models suboptimal
Outlook

- Investigations on differences in atmospheric models
- Use of supplementary data, e.g. CHAMP occultations
- Estimation of errors of gravity variation from models
- Implementation of error propagation to gravity field analysis
- Influence of sampling
Fig. 3 – Latitude dependence of the gravity corrections
C00 and C20 in geoid height [m]